Indian Statistical Institute
Back Paper Examination
Differential Topology: MMath II

Max Marks: 100 Time: 3 hours

(1) Define the term : embedding. Check that the map $f : \mathbb{R} \longrightarrow \mathbb{R}^2$ defined by

$$f(t) = \left(\frac{e^t + e^{-t}}{2}, \frac{e^t - e^{-t}}{2} \right)$$

is an embedding. [10]

(2) Let $X, Y \subseteq \mathbb{R}^6$ be the subsets defined by the equations

$$x_1^2 + x_2^2 + x_3^2 - x_4^2 = 1,$$
$$x_4^2 - x_5^2 - x_6^2 = -1.$$

respectively. Prove that X, Y are manifolds. Find their dimensions. Show that X and Y intersect transversally. [15]

(3) State the stability theorem. Show by examples that the properties (i) being an immersion, (ii) being transverse to a given closed submanifold are not stable properties of maps defined on noncompact domains. [15]

(4) Let X, Z be submanifolds of Y. Prove that if $X \pitchfork Z$, then for all $y \in (X \cap Z)$ we have

$$T_y(X \cap Z) = T_y(X) \cap T_y(Z).$$

[10]

(5) Let X be a manifold. Show that the tangent bundle $T(X)$ of X is a manifold. [10]

(6) Discuss the definition of the intersection number $I_2(f, Z)$ where $f : X \longrightarrow Y$ is a smooth map and Z a submanifold of Y. State all your assumptions clearly. If $f : X \longrightarrow Y$ is homotopic to a constant map show that $I_2(f, Z) = 0$ for all submanifolds of Z of Y that are closed in Y and of complementary dimension except perhaps if X has dimension zero. [15]

(7) Prove that S^2 and the torus are not diffeomorphic. [10]

(8) Prove that $H^1(\mathbb{R}^2 - 0) \neq 0$. [15]